A New Analytic Design Technique for Two- and Three-Way Warped Mode Combline Directional Couplers

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Abstract —An analytic technique of designing ultra-wide-band warped-mode two-way and three-way microwave directional couplers is presented. With this matrix method it is possible to design couplers for any level of coupling by choosing a warping angle appropriate to the desired output amplitudes. The design technique involves obtaining a "wave propagation" matrix in analytical form as a function of the "taper parameter," which in turn is a function of distance along the coupler length. The line parameters and the forward scattering matrix are then determined from the wave propagation matrix at any distance from the beginning of the coupler. The method is demonstrated here for couplers using microstrip coupled comblines. The computed and the measured performance of a two-way and a three-way microstrip combline coupler are presented for equal power splitting. These couplers were found to work between 1.5 and 8.5 GHz.

I. Introduction

A DIRECTIONAL coupler having unaltered line parameters and coupling coefficients along its length depends on the interference of normal modes [3], [4] for power transfer from the excited line to the coupled line or lines. Because of this, such a coupler may be called a mode interference coupler [3], [11]. In contrast to this, it is possible to vary simultaneously the line parameters and the coupling coefficients along the coupler length so that there is no interference of the normal modes while only one of the normal modes propagates. Such a technique was referred to by Fox [2] (for a two-guide coupler) as normal mode warping. A coupler employing this technique for power transfer was called by Fox [2] and Louisell [3] a tapered mode coupler and by Gunton [5] a warped mode coupler. It may be mentioned here that the discussions and analyses presented by Fox [2], Louisell [3], and Gunton [5] were for two-way couplers only.

Due to the interference of normal modes, the bandwidth of a single section mode interference coupler [11] is usually narrow (nearly an octave). The advantage of a warped mode coupler over a mode interference coupler is its large bandwidth, as stated by the previous authors [1]–[3]. The multioctave bandwidth of warped mode couplers can be seen from the experimental results presented in [5], [6], and [8]. The design of such a coupler for realization in planar microstrip lines has been considered in [5], [6], and [8].

For a two-line warped mode coupler, Louisell [3] suggested two ways of reducing "mode crosstalk" [3] and named the couplers achieved using these two methods as (i) uniform single tapered mode couplers and (ii) constant local beat wavelength couplers. Gunton [5] investigated two-line couplers of this second type. In this constant local beat wavelength type of coupler, the line parameters are varied in such a way that the required eigenmode is obtained while a constant difference between the normal mode propagation constants of the adjacent lines is maintained throughout the length of a coupler. The present discussion is on this type of warped mode coupler.

Gunton [5] implemented the analysis of Louisell [3] in making a two-way warped mode coupler using comblines [10], [5] in triplate microstrip configuration. His [5] experimental results on the constant local beat wavelength warped mode coupler using comblines were very encouraging. Herscher and Carroll [6] examined one example of normal mode warping in a three-way combline coupler of the constant local beat wavelength type. In this work a warped mode coupler was approximated as several tandem connected "uniform combline directional couplers" (UCDC's) [11] of equal length. These results motivated the author to search for a straightforward analytic technique for designing warped mode combline directional couplers (WMCDC's).

This paper presents a design technique which allows one to design systematically two- and three-line WMCDC's. The method is simple, easy to use, and capable of producing couplers with arbitrary power distribution between the excited line and the coupled line or lines. By using a nonlinear variation of the "taper parameter," the ripples in the characteristics, especially the initial overshoots and undershoots, are reduced to a negligible level.

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II. Normal Mode Equations for Mode Interference and Warped Mode Couplers

Consider a coupler consisting of N coupled transmission lines. The voltages $v_1, v_2, \cdots, v_N$ and currents $i_1, i_2, \cdots, i_N$ on the lines may be represented by vectors $\boldsymbol{v}$ and $\boldsymbol{i}$, respectively. Taking z as the distance along the length of the coupler, the equation for these coupled lines may be written as

$$\frac{\partial \boldsymbol{v}}{\partial z} = -j\boldsymbol{X}\boldsymbol{i} = -j\omega \boldsymbol{L}\boldsymbol{i} \tag{1a}$$

$$\frac{\partial \boldsymbol{i}}{\partial z} = -j\boldsymbol{B}\boldsymbol{v} = j\omega \boldsymbol{C}\boldsymbol{v}. \tag{1b}$$

Here, the matrices $\boldsymbol{X}$ and $\boldsymbol{B}$ are tridiagonal; this is because in a microstrip planar structure coupling exists between adjacent neighbors only and any coupling beyond nearest neighbors can be neglected. For comblines the off-diagonal (coupling) terms of $\boldsymbol{X}$ (or the inductance matrix $\boldsymbol{L}$) are zero since the coupling between two comblines is predominantly capacitive [5].

By introducing a termination matrix $\boldsymbol{R}_t$ [7] such that

$$\boldsymbol{v} = \boldsymbol{R}_t \boldsymbol{i} \quad \text{or} \quad \boldsymbol{R}_t^{-1/2}\boldsymbol{v} = \boldsymbol{R}_t^{1/2}\boldsymbol{i} \tag{2}$$

and by defining the forward wave amplitude vector $\boldsymbol{a}$ as

$$\boldsymbol{a} = \left(\boldsymbol{R}_t^{-1/2}\boldsymbol{v} + \boldsymbol{R}_t^{1/2}\boldsymbol{i}\right)/2 \tag{3}$$

the following equation may be obtained from (1a) and (1b):

$$\frac{\partial \boldsymbol{a}(z)}{\partial z} = -j\boldsymbol{J}\boldsymbol{a}(z). \tag{4}$$

Here,

$$\boldsymbol{J} = \left(\boldsymbol{R}_t^{-1/2}\boldsymbol{X}\boldsymbol{R}_t^{-1/2} + \boldsymbol{R}_t^{1/2}\boldsymbol{B}\boldsymbol{R}_t^{1/2}\right)/2 \tag{5}$$

is the symmetric wave propagation matrix which is tridiagonal and contains necessary information for physical realization of the coupled lines of a coupler [11].

From (4) one can write

$$\frac{\partial \boldsymbol{w}(z)}{\partial z} = -j\boldsymbol{\beta}\boldsymbol{w}(z) \tag{6}$$

where

$$\boldsymbol{w}(z) = \boldsymbol{Q}\boldsymbol{a}(z) = \text{normal mode amplitude vector} \tag{7a}$$

and

$$\boldsymbol{\beta} = \boldsymbol{Q}\boldsymbol{J}\boldsymbol{Q}^t = \text{a diagonal matrix}. \tag{7b}$$

The diagonal elements of the matrix $\boldsymbol{\beta}$ are the eigenvalues of $\boldsymbol{J}$ (normal mode propagation constants). The matrix $\boldsymbol{Q}$ is orthogonal so that $\boldsymbol{Q}\boldsymbol{Q}^t = \boldsymbol{I}$ (identity matrix). The row vectors of the $\boldsymbol{Q}$ matrix are $\boldsymbol{q}_i^t$ $(i = 1, 2, \cdots N)$, the orthonormal eigenvectors of the $\boldsymbol{J}$ matrix. Here $\boldsymbol{q}_k$ is the eigenvector corresponding to the eigen–propagation constant β_k.

From (6) we get

$$\boldsymbol{w}(z) = \exp(-jz\boldsymbol{\beta})\boldsymbol{w}(0), \qquad \text{where } \boldsymbol{w}(0) = \boldsymbol{Q}\boldsymbol{a}(0). \tag{8}$$

From (7a) and (8) the forward wave amplitude vector in terms of the normal mode propagation constants may be written as

$$\begin{aligned}\boldsymbol{a}(z) &= \boldsymbol{Q}^t\boldsymbol{w}(z) = \boldsymbol{Q}^t\exp(-jz\boldsymbol{\beta})\boldsymbol{Q}\boldsymbol{Q}^t\boldsymbol{w}(0)\\ &= \boldsymbol{Q}^t\exp(-jz\boldsymbol{\beta})\boldsymbol{Q}\boldsymbol{a}(0).\end{aligned} \tag{9}$$

It is usual practice to write the equation for the power of each line of a coupler in terms of the wave amplitudes. However, expressing power in terms of the normal mode amplitudes [9] helps in understanding how coupling occurs due to interference of the normal modes. Here (9) is the forward wave amplitude equation, in general, for a uniform mode interference coupler where $\boldsymbol{J}$ is not a function of z and all the normal modes are excited.

Now in a warped mode coupler, at the input side, only one of the normal modes is excited by exciting a single input port. The $\boldsymbol{J}$ matrix is then varied along the coupler length in such a way that only the excited normal mode propagates. This means that the impedance and the phase velocity of individual lines and the interline coupling capacitances vary along the length. Variation of the $\boldsymbol{J}$ matrix also means that the $\boldsymbol{Q}$ matrix, the $\boldsymbol{a}$ vector, and the $\boldsymbol{w}$ vector change with distance. Therefore, in such a case (6) would take the form [8]

$$\frac{\partial \boldsymbol{w}(z)}{\partial z} = -j\boldsymbol{\beta}(z)\boldsymbol{w}(z) - \boldsymbol{Q}(z)\frac{\partial \boldsymbol{Q}(z)}{\partial z}\boldsymbol{w}(z). \tag{10}$$

From (10) it may be observed that the solution for $\boldsymbol{w}(z)$ is no longer as simple as (8). A general solution of $\boldsymbol{w}(z)$ in matrix form was shown in [8].

III. The Design Problem

Consider exciting one of the normal modes by exciting the input of line 2 of a warped mode coupler ($a_2(0) = 1$) so that at the input (at $z = 0$) the coupling is zero. This means at $z = 0$ the $\boldsymbol{Q}$ is an identity matrix. Using (7a) we can write at $z = 0$

$$\boldsymbol{a}(0) = \boldsymbol{w}(0) = \boldsymbol{q}_2(0). \tag{11}$$

Now consider that the $\boldsymbol{J}$ matrix is varied in such a way that propagation of the single normal mode is maintained. It is assumed that this is done in such a way that it is possible to force the solution of (10) to approximately take the form of (8).

Such an ideal mode warping results in extra wide band performance of the couplers due to the absence of interference of the normal modes. For this warped mode coupler one may write the forward wave amplitude vector at the output (at $z = L$) as

$$\begin{aligned}\boldsymbol{a}^t(L) &= w_2(0)\exp(-j\beta_2 z)\boldsymbol{q}_2^t(L)\\ &= \exp(-j\beta_2 z)\boldsymbol{q}_2^t(L).\end{aligned} \tag{12}$$

At this stage the mode warping may be seen as vector rotation as shown in Fig. 1. Now, from the ith element of the forward wave amplitude vector $\boldsymbol{a}(L)$ the power at the output of the ith line may be obtained using

$$P_o^i(L) = a_i^*(L)a_i(L). \tag{13}$$

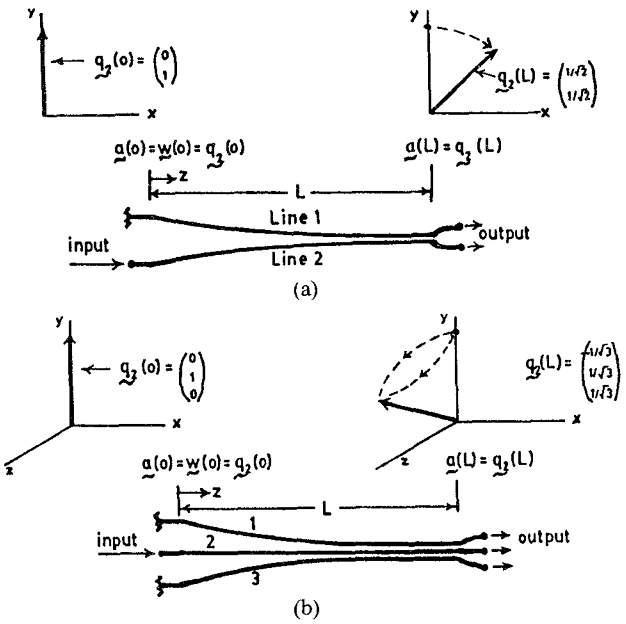

Fig. 1. Forward wave amplitudes and the normal-mode vector at the input and output of (a) a two-way and (b) a three-way warped mode coupler. $\beta_2 = 0$ is assumed for both the cases so that $a(L) = q_2(L)$. Also, equal power splitting has been assumed for both cases.

Equation (10) indicates that it is not a simple task to obtain such an ideal mode warping. Previous work [3], [5] on two-line warped mode couplers had indicated that although one of the normal modes was excited, the output power expression at the end of the coupler contained a contribution from the other normal mode in addition to the major contribution from the excited normal mode [3], [9]. Such contributions from other normal modes, appearing as a consequence of "hypercoupling" [3] between the normal modes, is termed mode crosstalk [3]. This causes undesirable ripples in the power versus frequency response of the coupler, causing deviation from the expected performance. The mode crosstalk can be minimized by following Louisell's [3] method of maintaining a constant difference between the normal mode propagation constants along the length and warping the mode slowly over a long distance.

The design problem then appears as one of obtaining a $\boldsymbol{J}$ matrix and having one of the normal mode vectors expressed in analytical terms. It is required that both the $\boldsymbol{J}$ matrix and the normal mode vector have a common varying parameter such that when that parameter is varied, $\boldsymbol{J}$ changes, producing the desired eigenvector as the eigenmode of propagation. The restriction of having a constant difference of normal mode propagation constants means that $(\beta_1 - \beta_2) = \text{const.}$ for a two-way coupler and $(\beta_1 - \beta_2) = (\beta_3 - \beta_2) = \text{const.}$ for a three-way coupler. Slow warping is an additional restriction.

IV. Multisection Approximation of a Warped Mode Coupler

For physical realization and for obtaining the power/phase versus frequency characteristics, a warped mode coupler may be approximated as a tandem connection of a large number of uniform coupler sections of equal length (L_0). This approximation is valid as long as the number of sections (P) is sufficiently large that the variation of the line parameters is almost continuous throughout the coupler. For each of these small sections the theory of UCDC [7], [11] may be applied. It has been observed that satisfactory results can be obtained by taking a value of $P = 10$. There is no need to increase this number beyond 30.

A. Computation of Theoretical Characteristics

Using the theory of uniform coupler [7], [11] the forward scattering matrix of the kth section ($\boldsymbol{S}_{0i}^k$) may be written as

$$\boldsymbol{S}_{0i}^k = \boldsymbol{Q}_k^t \exp\{-jL_0(f/f_1)\boldsymbol{\beta}\}\boldsymbol{Q}_k \tag{14}$$

where L_0 is the coupled length of each section $= L/P$, f is the frequency of computation, f_1 is the normalizing frequency, and the $\boldsymbol{\beta}$ matrix is as defined in (7b). From (14) it may be observed that the computation of $\boldsymbol{S}_{0i}^k$ may be done if the wave propagation matrix of the kth section, $\boldsymbol{J}_k$, is known.

Since the propagation of coupled waves in a combline coupler has been assumed to be in the forward direction (with negligible propagation in the opposite direction), the forward scattering matrix of the total coupler may be written as

$$\boldsymbol{S}_{0i} = \boldsymbol{S}_{0i}^P \cdots \boldsymbol{S}_{0i}^2 \boldsymbol{S}_{0i}^1 \tag{15}$$

where $\boldsymbol{S}_{0i}^k$ ($k = 1, 2, 3, \cdots, P$) is the scattering matrix of the kth section of the coupler.

The amplitude at the output ports is then obtained by using the relationship

$$\boldsymbol{b}_o = \boldsymbol{S}_{0i}\boldsymbol{a}_i \tag{16}$$

where $\boldsymbol{b}_o$ is the output wave amplitude vector and $\boldsymbol{a}_i$ is the input wave amplitude vector. From the complex amplitude vector $\boldsymbol{b}_o$ the power and relative phases of the output ports are obtained.

B. Obtaining Parameter Values and Dimensions from the Wave Propagation Matrices

An analytical form of the $\boldsymbol{J}$ matrix of a warped mode coupler as a function of z would permit computation of the numerical values of the elements of $\boldsymbol{J}$ at the middle of the P sections along the coupler length. The inductance ($\boldsymbol{L}$) and capacitance ($\boldsymbol{C}$) matrices for each section are then computed from the $\boldsymbol{J}$ matrix of the corresponding section by using the technique for UCDC's shown in [11]. From the $\boldsymbol{L}$ and $\boldsymbol{C}$ matrices the impedances, phase velocities of the transmission lines, and coupling capacitances are computed using the equations shown in [11]. Using these values the dimensions of the comblines are determined from a model of comblines [12], [9] and the amount of finger overlap is determined from the experimental curves [12], [9]. The usual length of such a coupler has been found to be of the order of three to four times the wavelength at the center frequency.

The technique of scaling the $\boldsymbol{J}$ matrices [8], [11] is used here to obtain physically realizable line parameters. This scaling does not affect the power characteristics. The scaling equation is

$$J = Q^t \beta Q = Q^t(\beta_{[1]} - \beta_0 I)Q = J_{[1]} - \beta_0 I \qquad (17)$$

where the subscript [1] is used to denote matrices before scaling.

V. A Technique for Obtaining the Wave Propagation Matrices for a Warped Mode Coupler

The technique utilizes the relationship

$$JQ^t = Q^t \beta \qquad (18)$$

which is obtained from equation (7b). For convenience the derivation procedures of the J matrices for two-way and three-way warped mode couplers are presented in Appendixes I and II.

A. Two-Way Warped Mode Coupler

For a two-way warped mode coupler the analytical form of matrices J, β, Q and the normal mode vector q_2^t are shown in (23), (20c), (24) and (22) of Appendix I.

For the alternative choice J, β, Q and q_2^t are as shown in (25a), (25b), (25c), and (25d) of Appendix I.

B. Three-Way Warped Mode Coupler

For a three-way warped mode coupler the analytical form of matrices J, β, and Q and the normal mode vector are as shown in (32a), (32b), (32c), and (32d) of Appendix II.

For the alternative choice J, β, Q, and q_2^t are as shown in (34a), (34b), (34c), and (34d) of Appendix II.

VI. Linear and Nonlinear Variation of the Taper Parameter

The variation of the "taper parameter" plays a significant role in reducing the initial overshoot and undershoot as well as the subsequent ripples along the frequency scale of the characteristics of a warped mode coupler. A discussion of this was presented in [8] and is valid for this work also.

In this work it was found that for both two-line and three-line warped mode couplers the following nonlinear taper gives the best result:

$$\sigma(z) = \left[\frac{z}{L} - \frac{1}{2\pi}\sin\left(\frac{\pi z}{L}\right)\right]\Phi \qquad (\text{for } z = 0 \text{ to } L) \quad (19a)$$

or

$$\sigma(k) = \left[\frac{(k-1/2)}{P} - \frac{1}{2\pi}\sin\left\{\pi\frac{(k-1/2)}{P}\right\}\right]\Phi \qquad (\text{for } k = 1,2,3,\cdots,P) \quad (19b)$$

where Φ = total warping angle, z = distance along the length of the coupler, and L = total length of the warped mode coupler. $\Phi = \cos^{-1}(1/\sqrt{2})$ for two-way equal power splitting and $\Phi = \cos^{-1}(1/\sqrt{3})$ for a three-way equal power splitting coupler.

Gunton [5] showed for a two-way warped mode coupler that instead of using the linear variation a $\sin^2$ variation [5] can reduce the ripples in the characteristics. Unfortunately the $\sin^2$ variation produces much larger initial overshoot in the characteristics. Compared to Gunton's results, the characteristics obtained here using the nonlinear variation presented (equation (19a) or (19b)) show a significant improvement in terms of initial overshoot as well as the subsequent ripples.

For each line of a coupler the theoretical power plot using the nonlinear variation of (19b) is superimposed on the plot using the linear variation of the taper parameter. The improvement in characteristics for a two-way and a three-way WMCDC may be observed in Figs. 2 and 3, respectively.

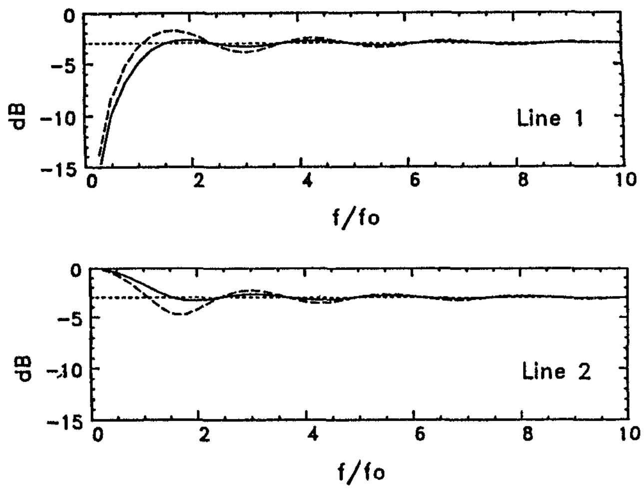

Fig. 2. Computed power (P_o^t/P_{in}) against frequency characteristics of a two-way warped mode coupler for linear and the best nonlinear variation of the taper parameter. ---- linear; —— nonlinear.

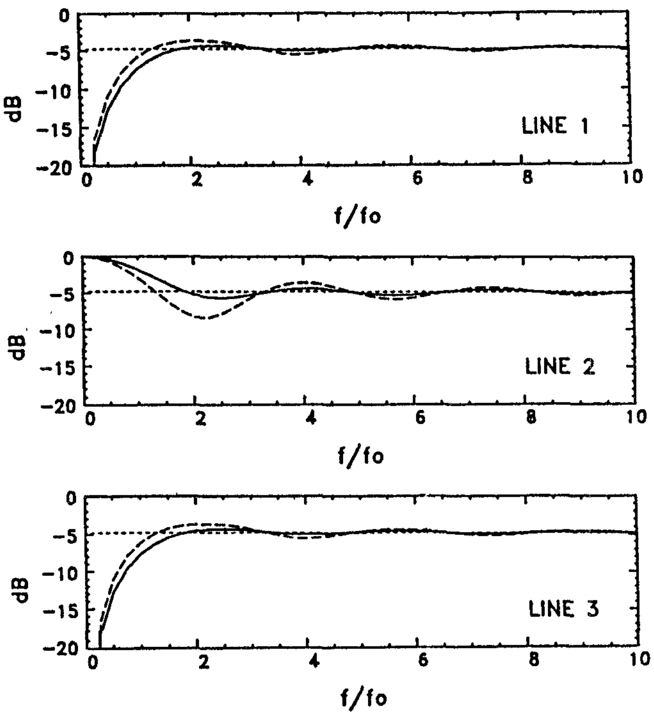

Fig. 3. Computed power (P_o^t/P_{in}) against frequency characteristics of a three-way warped mode coupler for linear and the best nonlinear variation of the taper parameter. ---- linear; —— nonlinear.

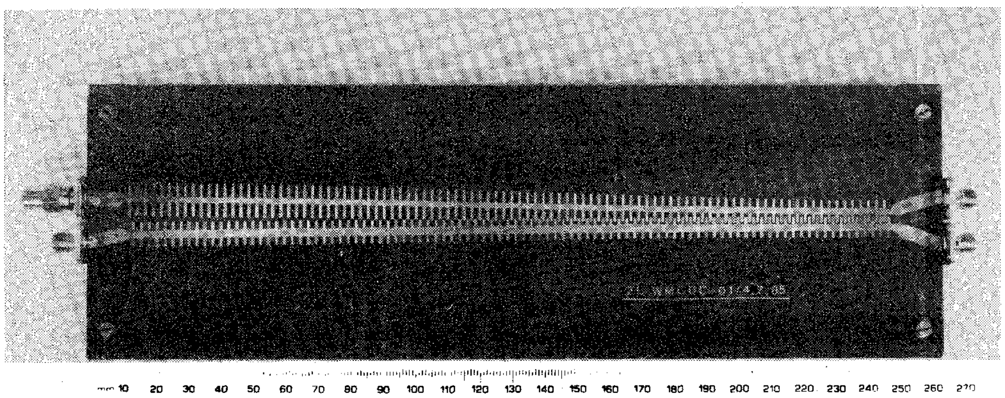

Fig. 4. Photograph of the experimental two-way WMCDC for coupling above 1.5 GHz.

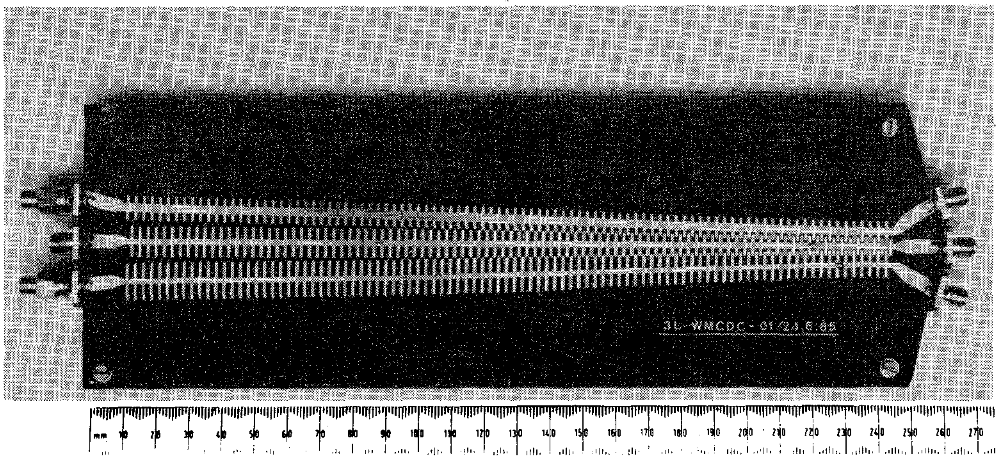

Fig. 5. Photograph of the experimental three-way WMCDC for coupling above 1.5 GHz.

VII. Design and Fabrication of WMCDC's

A. Design Examples of WMCDC's

For both two-way and three-way WMCDC's the lower band-edge frequency was chosen to be 1.5 GHz for convenience of fabrication and measurement using the available facilities. The nonlinear variation of the taper parameter of (19b) was used in the design of both of these couplers.

An appropriate value of $\Delta\beta$ was chosen so that the coupled length and the line parameters are physically realizable and none of the off-diagonal terms of $\boldsymbol{J}$ appears as a positive number at any position along the coupler length. The important factors regarding physical realizability which need attention are the unattainable values of phase velocity spreads of the individual lines and the unattainable values of interline coupling capacitances.

The parameter values which were the same for both couplers are: coupled length $L = 228$ mm, finger periodicity $= 2.4$ mm, dielectric constant $\epsilon_r = 2.55$, thickness of dielectric substrate $h = 1.5$ mm, finger line impedance $Z_f = 107\ \Omega$, and coupled combline impedance $= 50\ \Omega$.

1) Two-Way WMCDC: The $\boldsymbol{J}$ matrix of (23) was chosen and the design was made for exciting the input port of line 2. For this example coupler the coupled length could have been reduced by 8 percent. In general, it was found that reducing the length causes difficulty in physical realization of the comblines and of finger overlaps between the adjacent lines.

2) Three-Way WMCDC: For this coupler the $\boldsymbol{J}$ matrix of (32a) was chosen. The design was made for exciting the input port of line 2.

B. Fabrication of the WMCDC's

For this purpose 1:1 masks on rubylith film were prepared using the computer-operated scribing machine and techniques described in [8]. The number of sections used was 10. The masks were then used to make couplers by using photoreversal and photolithographic techniques from a copper-cladded dielectric substrate (3M GX-0600-55-11, $\epsilon_r = 2.55$).

The fabricated two-way WMCDC is shown in Fig. 4 and the fabricated three-way WMCDC is shown in Fig. 5.

VIII. Measured Results of the Experimental WMCDC's

A. Two-Way WMCDC

The computed and measured power characteristics of the experimental two-way coupler are shown in Fig. 6(a) and (b). The relative phase response of line 1 with respect to line 2 is shown in Fig. 6(c). From the power characteristics it may be observed that the coupler works well within

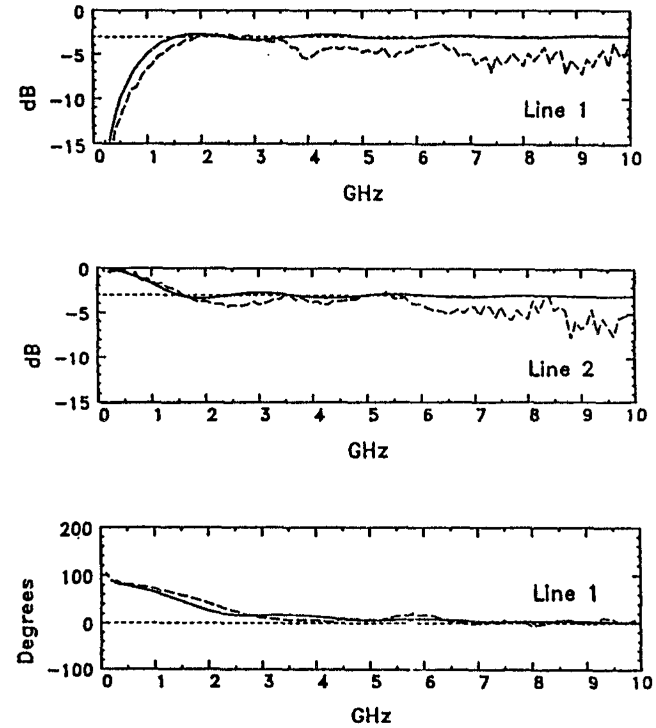

Fig. 6. Measured power and relative phase characteristics of the two-way WMCDC shown in Fig. 5. Computed characteristics are superimposed for comparison. ——— computed; ---- measured.

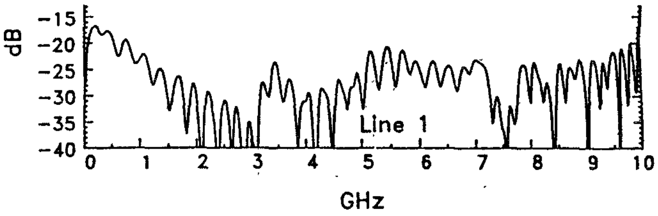

Fig. 7. Measured power characteristics at the backport of line 1 of the two-way WMCDC.

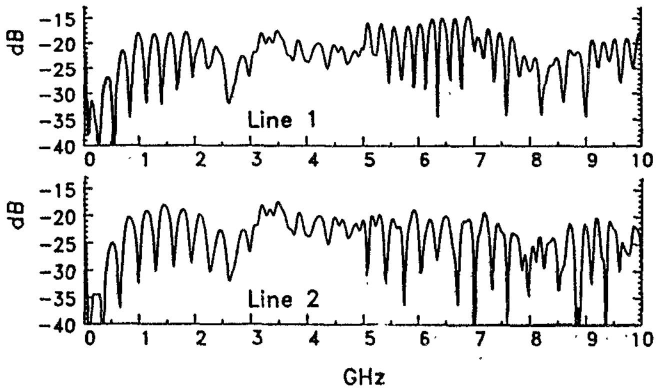

Fig. 8. Measured return loss of the lines of the two-way WMCDC.

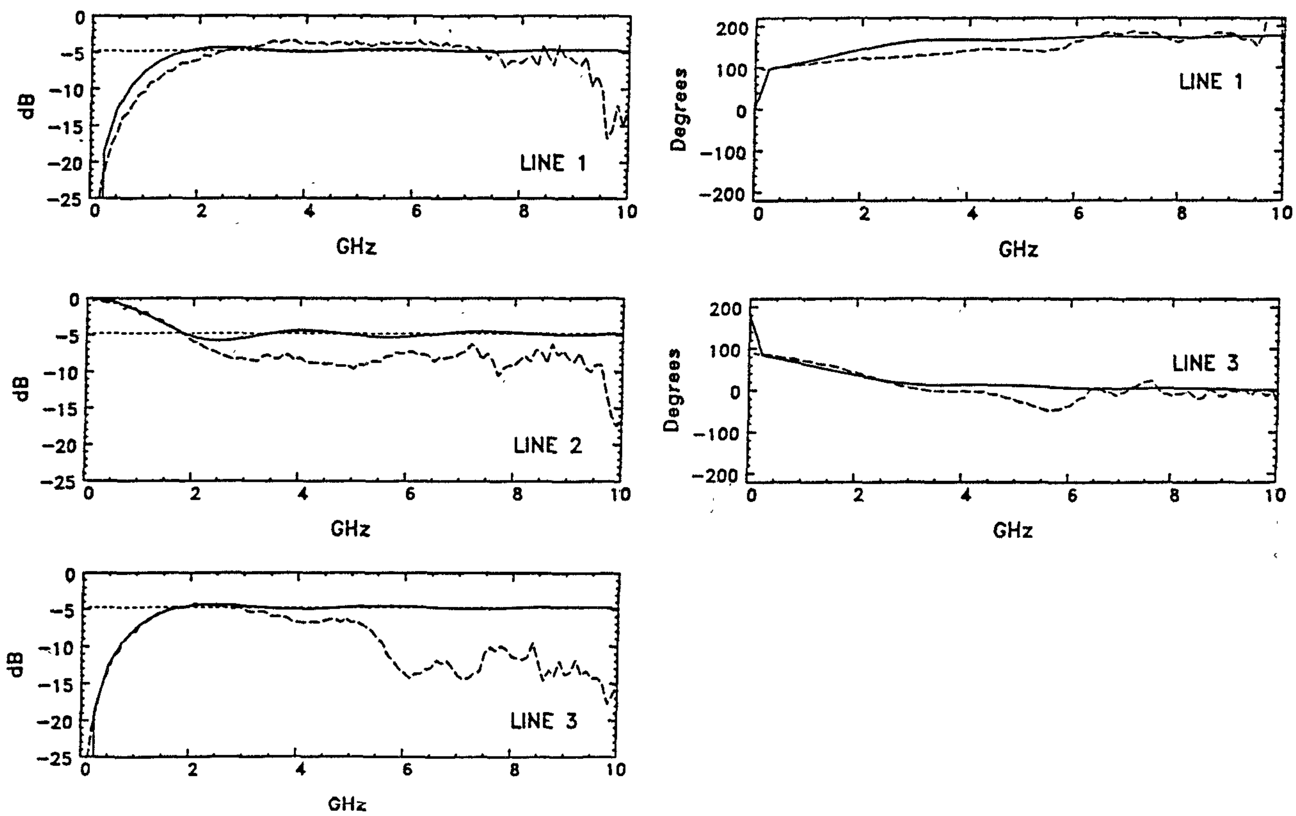

Fig. 9. Measured power and relative phase characteristics of the three-way WMCDC shown in Fig. 6. Computed characteristics are superimposed for comparison. ——— computed; ---- measured.

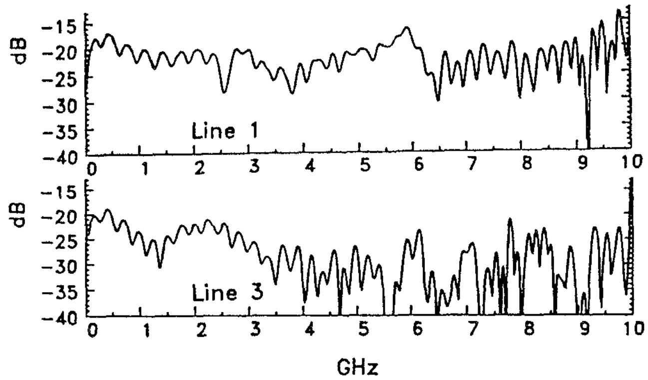

Fig. 10. Measured power characteristics at the backports of line 1 and line 3 of the three-way WMCDC.

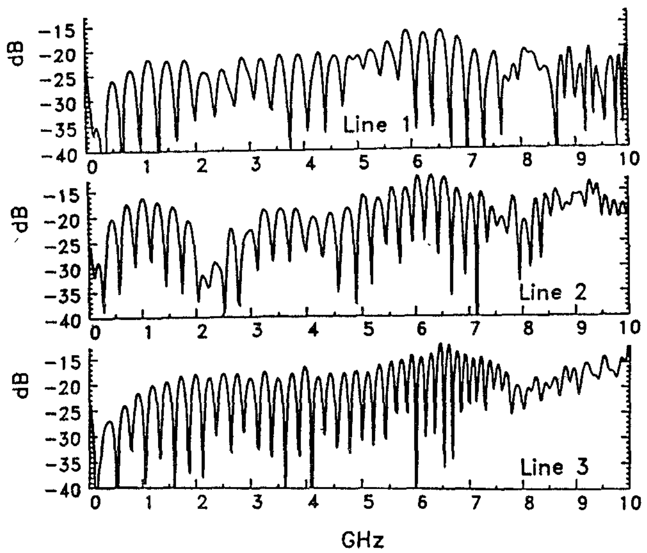

Fig. 11. Measured return loss of the lines of the three-way WMCDC.

the frequency range 1.5–8.5 GHz. The phase response of line 1 with respect to line 2 shows very good agreement.

The power characteristic at the backport of line 1 is shown in Fig. 7 while the return loss of the two lines are shown in Fig. 8.

B. Three-Way WMCDC

The computed and measured power characteristics of the fabricated three-way coupler are shown in Fig. 9(a)–(c). The relative phase characteristics are shown in Fig. 10(d). The power characteristics of line 1 show good agreement up to 8.5 GHz. Line 1 shows overcoupling whereas line 3 shows undercoupling. Unfortunately line 3 shows excessive loss after 5 GHz, which is thought to be due to its long fingers as well as incorrect phase velocity. It has been observed that a small error in the phase velocity of a coupled line causes significant deviation in performance. The probable causes of the observed power imbalance are errors in the realized phase velocities and in coupling capacitances.

The power characteristics at the backports of line 1 and line 3 are shown in Fig. 10. Fig. 11 shows the return loss of the lines.

IX. Conclusions

Here, due to the exact analytical expressions of the elements of the wave propagation matrix $\boldsymbol{J}$ and the modal matrix $\boldsymbol{Q}$, the computed values are exact. In the technique presented here, the restriction on the output power splitting ratio is that it be in accordance with the analytical expressions of the normal mode vectors presented. For the three-way warped mode coupler, symmetry in the power levels of line 1 and line 3 has resulted because of the chosen form of the desired normal mode vector.

Under ideal condition there should be no limitation on the bandwidth of this type of coupler. Practically the finger resonance of the transmission lines (i.e., the comblines) of a WMCDC imposes a restriction on the upper limit of the operating band.

An important feature of these WMCDC's is that a high level of coupling is possible while maintaining the same wide bandwidth. The relative phase of the output ports of a warped mode coupler is either 0° or 180° and is constant within the band of performance.

This design method does not require any optimization; as a result the design computation can be done in a short time using a personal computer or a desktop computer. The method presented here is for designing and constructing couplers with microstrip comblines. However, the same technique may be used for realizing couplers in any other planar structure capable of producing forward coupling.

Appendix I
Wave Propagation Matrices for a Two-Way Warped Mode Coupler

For a two-way warped mode coupler the wave propagation matrix $\boldsymbol{J}$, the eigen-propagation constant matrix $\boldsymbol{\beta}$ and the eigenvector matrix $\boldsymbol{Q}$ may be written as

$$\boldsymbol{J}=\begin{bmatrix} a & f \\ f & b \end{bmatrix} \tag{20a}$$

$$\boldsymbol{\beta}_{[1]}=\begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \tag{20b}$$

$$\boldsymbol{\beta}=\begin{bmatrix} \Delta\beta & 0 \\ 0 & 0 \end{bmatrix} \tag{20c}$$

$$\boldsymbol{Q}=\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}=\begin{bmatrix} -\boldsymbol{q}_1^t- \\ -\boldsymbol{q}_2^t- \end{bmatrix}. \tag{20d}$$

For a warped mode coupler the $\boldsymbol{J}$ and $\boldsymbol{Q}$ matrices are functions of distance z. Here, $\Delta\beta=(\beta_1-\beta_2)$ and (20c) is obtained from (20b) by using the scaling technique described earlier. The reason for making $\beta_2=0$ is that $\boldsymbol{q}_2^t=(q_{21}, q_{22})$ has been taken as the desired normal mode eigenvector by exciting the input port of line 2.

Using the $\boldsymbol{J}$ of (20a), the Q of (20b), and the $\boldsymbol{\beta}$ of (20c) in (18) and equating the second column (since $\beta_2=0$) of both the sides, the following equations are obtained:

$$aq_{21} + fq_{22} = 0 \tag{21a}$$

$$fq_{21} + bq_{22} = 0. \tag{21b}$$

We choose the desired normal mode vector as

$$\boldsymbol{q}_2^t = (S, C) \qquad \text{where } S = \sin\sigma \text{ and } C = \cos\sigma. \tag{22}$$

Thus the output of line 1 is in phase with that of line 2. For complete power transfer $\sigma(L) = \pi/2$, while for equal power splitting $\sigma(L) = \pi/4$.

Now, choosing $a = \Delta\beta C^2$, where $\Delta\beta$ is any constant independent of σ, we get $f = -\Delta\beta CS$ from (21a) and $b = \Delta\beta S^2$ from (21b). Putting the values of a, b, and f in (20a) we get

$$\boldsymbol{J} = \Delta\beta\begin{bmatrix} C^2 & -CS \\ -CS & S^2 \end{bmatrix}. \tag{23}$$

As can be seen, the off-diagonal terms of the $\boldsymbol{J}$ matrix are negative, which means that the coupling between the lines is capacitive. This is necessary for realizing a coupler using comblines.

From (23) the two eigenvalues are $\lambda_1 = \Delta\beta$ and $\lambda_2 = 0$. The eigenvalues λ_1 and λ_2 are independent of σ. As a result, while $\boldsymbol{J}$ and $\boldsymbol{q}_2^t$ change with z, the eigenvalues remain constant as $(\Delta\beta, 0)$, which satisfies the condition for the intended type of warped mode coupler [3]. Using the two eigenvalues one can obtain the two eigenvectors and form the $\boldsymbol{Q}$ matrix as

$$\boldsymbol{Q} = \begin{bmatrix} C & -S \\ S & C \end{bmatrix}. \tag{24}$$

Now, by choosing appropriate value of $\sigma(L)$ it is possible to have any amount of power transfer as long as the physical dimensions remain within a realizable range.

As an alternative to the choice of $\boldsymbol{q}_2^t$ of (22) one may choose $q_2^t = (-S, C)$ with excitation at the input of line 2. In this choice the output of the first line will be 180° out of phase with that of the second line. The matrices $\boldsymbol{J}$, $\boldsymbol{\beta}$, and $\boldsymbol{Q}$ and the vector $\boldsymbol{q}_2^t$ for this alternative choice are

$$\boldsymbol{J} = \Delta\beta\begin{bmatrix} S^2 & -CS \\ -CS & C^2 \end{bmatrix} \tag{25a}$$

$$\boldsymbol{\beta} = \begin{bmatrix} 0 & 0 \\ 0 & \Delta\beta \end{bmatrix} \tag{25b}$$

$$\boldsymbol{Q} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix} \tag{25c}$$

$$\boldsymbol{q}_2^t = (-S, C). \tag{25d}$$

Appendix II
Wave-Propagation Matrices for a Three Way Warped Mode Coupler

For a three-way warped mode coupler in planar structure (nearest neighbor coupling), the $\boldsymbol{J}$ and $\boldsymbol{\beta}$ matrices may be assumed to be of the following form:

$$\boldsymbol{J} = \begin{bmatrix} a & f & 0 \\ f & b & g \\ 0 & g & c \end{bmatrix} \tag{26a}$$

$$\boldsymbol{Q} = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} \tag{26b}$$

$$\boldsymbol{\beta}_{[1]} = \begin{bmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \beta_3 \end{bmatrix} \tag{26c}$$

$$\boldsymbol{\beta} = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B_2 \end{bmatrix}. \tag{26d}$$

Here, the input of line 2 is chosen for excitation so that $\boldsymbol{q}_2^t$ is the desired eigenvector. The matrix $\boldsymbol{\beta}$ is scaled so that $\beta_2 = 0$. Using (26a) and (26d) in (18), and equating the middle (second) column of both sides, the following equations are obtained:

$$aq_{21} + fq_{22} = 0 \tag{27a}$$

$$fq_{21} + bq_{22} + gq_{23} = 0 \tag{27b}$$

$$gq_{22} + cq_{23} = 0. \tag{27c}$$

The desired normal mode eigenvector is chosen as

$$\boldsymbol{q}_2^t = (-S_1S_2, C_2, +S_2C_1) \tag{28}$$

with a cosine term in the middle, where $S_1 = \sin\sigma_1$, $S_2 = \sin\sigma_2$, $C_1 = \cos\sigma_1$, and $C_2 = \cos\sigma_2$. Now putting the values of q_{21}, q_{22}, and q_{23} from (28) in (27a), (27b), and (27c) and assuming $a = -\Delta\beta C_2$ and $c = \beta C_2$, where $\Delta\beta$ is any constant, the following $\boldsymbol{J}$ matrix is obtained:

$$\boldsymbol{J} = \Delta\beta\begin{bmatrix} -C_2 & -S_1S_2 & 0 \\ -S_1S_2 & (S_2^2C_1^2 - S_1^2S_2^2)/C_2 & -S_2C_1 \\ 0 & -S_2C_1 & C_2 \end{bmatrix}. \tag{29}$$

Solving the characteristic equation of this $\boldsymbol{J}$ matrix, the three eigenvalues are obtained as

$$\lambda_1 = \frac{1}{2}\Big[\Delta\beta(S_2^2C_1^2 - S_1^2S_2^2)/C_2 - \surd\Big\{\big(\Delta\beta(S_2^2C_1^2 - S_1^2S_2^2)/C_2\big)^2 + 4(\Delta\beta)^2\Big\}\Big]$$

$$\lambda_2 = 0$$

and

$$\lambda_3 = \frac{1}{2}\Big[\Delta\beta(S_2^2C_1^2 - S_1^2S_2^2)/C_2 + \surd\Big\{\big(\Delta\beta(S_2^2C_1^2 - S_1^2S_2^2)/C_2\big)^2 + 4(\Delta\beta)^2\Big\}\Big] \tag{30}$$

From (30) it is seen that in order to make these eigenvalues independent of σ_1 and σ_2, it is necessary to have $C_1 = S_1 = 1/\surd 2$. Applying this condition and taking $\Delta\beta = -(\beta_1 - \beta_2) = +(\beta_3 - \beta_2)$, the three eigenvalues may be written as $\lambda_1 = (\beta_1 - \beta_2)$, $\lambda_2 = 0$, and $\lambda_3 = (\beta_3 - \beta_2)$. For simplification we shall now replace C_2 by C and S_2 by S.

The $\boldsymbol{J}$ matrix, $\boldsymbol{\beta}$ matrix, and $\boldsymbol{Q}$ matrix and the vector $\boldsymbol{q}_2^t$ may now be written as

$$\boldsymbol{J}=\Delta\beta\begin{bmatrix} -C & -S/\sqrt{2} & 0 \\ -S/\sqrt{2} & 0 & -S/\sqrt{2} \\ 0 & -S/\sqrt{2} & C \end{bmatrix} \quad (31a)$$

$$\beta=\begin{bmatrix} -\Delta\beta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & +\Delta\beta \end{bmatrix} \quad (31b)$$

$$\boldsymbol{Q}=\begin{bmatrix} (C+1)/2 & S/\sqrt{2} & (-C+1)/2 \\ -S/\sqrt{2} & C & S/\sqrt{2} \\ (-C+1)/2 & -S/\sqrt{2} & (C+1)/2 \end{bmatrix} \quad (31c)$$

$$\boldsymbol{q}_2^t=(-S/\sqrt{2}, C, +S/\sqrt{2}). \quad (31d)$$

One can see that the normal mode vector $\boldsymbol{q}_2^t$ (relevant to the desired normal mode amplitude w_2) as shown above, warps from $\boldsymbol{q}_2^t=(0,1,0)$ to the desired level, e.g., $\boldsymbol{q}_2^t=(-1/\sqrt{3},1/\sqrt{3},1/\sqrt{3})$ if an equal power splitting is desired.

From the expression of $\boldsymbol{q}_2^t$ presented in (31d) it is seen that the first element of $\boldsymbol{q}_2^t$ is negative while the other two are positive. This indicates that with respect to the output of the middle line the output of line 1 is 180° out of phase, while the output of line 3 is in phase.

As an alternative to this it is possible to have the $\boldsymbol{q}_2^t$ as

$$\boldsymbol{q}_2^t=(+S/\sqrt{2}, C, -S/\sqrt{2}). \quad (32)$$

In this case it is necessary to choose $a=+\Delta\beta C_2$ and $b=-\Delta\beta C_2$ in order to have the $\boldsymbol{J}$ matrix with negative off-diagonal elements. The $\boldsymbol{J}$, $\boldsymbol{\beta}$, and $\boldsymbol{Q}$ matrices for this choice are

$$\boldsymbol{J}=\Delta\beta\begin{bmatrix} C & -S/\sqrt{2} & 0 \\ -S/\sqrt{2} & 0 & -S/\sqrt{2} \\ 0 & -S/\sqrt{2} & -C \end{bmatrix} \quad (33a)$$

$$\boldsymbol{\beta}=\begin{bmatrix} +\Delta\beta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\Delta\beta \end{bmatrix} \quad (33b)$$

$$\boldsymbol{Q}=\begin{bmatrix} (C+1)/2 & -S/\sqrt{2} & (-C+1)/2 \\ S/\sqrt{2} & C & -S/\sqrt{2} \\ (-C+1)/2 & S/\sqrt{2} & (C+1)/2 \end{bmatrix} \quad (33c)$$

$$\boldsymbol{q}_2^t=(+S/\sqrt{2}, C, -S/\sqrt{2}). \quad (33d)$$

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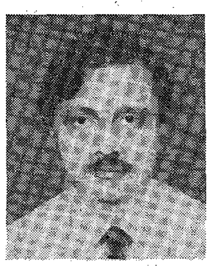

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